Exercise 1.1.5 - LCM and HCF Solutions

1. Find the LCM and the HCF of 10 and 15

Finding the HCF (Highest Common Factor)

First, let's list all the factors of each number:

Factors of 10: 1, 2, 5, 10
Factors of 15: 1, 3, 5, 15

The common factors are 1 and 5.
The highest common factor (HCF) is 5.

Finding the LCM (Least Common Multiple)

Method 1: List the multiples
Multiples of 10: 10, 20, 30, 40, 50, 60, ...
Multiples of 15: 15, 30, 45, 60, ...

The common multiples are 30, 60, ...
The least common multiple (LCM) is 30.

Method 2: Using prime factorization
$10 = 2 \times 5$
$15 = 3 \times 5$

$LCM = (2 \times 3 \times 5) = 30$

Therefore, HCF(10, 15) = 5 and LCM(10, 15) = 30

2. Find the LCM and the HCF of 24, 36, and 480

Finding the HCF

Using prime factorization:
$24 = 2^3 \times 3^1$
$36 = 2^2 \times 3^2$
$480 = 2^5 \times 3^1 \times 5^1$

The common prime factors are 2 and 3.
Taking the minimum power of each common prime factor:
$HCF = 2^2 \times 3^1 = 4 \times 3 = 12$

Finding the LCM

Using prime factorization, take the maximum power of each prime factor:
$LCM = 2^5 \times 3^2 \times 5^1 = 32 \times 9 \times 5 = 1440$

Therefore, HCF(24, 36, 480) = 12 and LCM(24, 36, 480) = 1440

3. Find the LCM and the HCF of $6a^2b^3$ and $12ab^2$

Finding the HCF

First, let's express each term with its numerical and variable factors:
$6a^2b^3 = 2^1 \times 3^1 \times a^2 \times b^3$
$12ab^2 = 2^2 \times 3^1 \times a^1 \times b^2$

Taking the minimum power of each common factor:
$HCF = 2^1 \times 3^1 \times a^1 \times b^2 = 6ab^2$

Finding the LCM

Taking the maximum power of each factor:
$LCM = 2^2 \times 3^1 \times a^2 \times b^3 = 12a^2b^3$

Therefore, $HCF(6a^2b^3, 12ab^2) = 6ab^2$ and $LCM(6a^2b^3, 12ab^2) = 12a^2b^3$

4. A gym trainer recommends that a client does cardio workouts every 3 days and strength training every 5 days. If the client does both cardio and strength training today, how many days will pass before their cardio and strength training sessions coincide again?

This is an application of the LCM concept. We need to find the LCM of 3 and 5.

Factors:
$3 = 3^1$
$5 = 5^1$

Since there are no common factors between 3 and 5, the LCM is simply their product:
$LCM(3, 5) = 3 \times 5 = 15$

Therefore, the client's cardio and strength training sessions will coincide again after 15 days.

5. A certain flower species blooms every 4 years, while a particular tree species produces fruit every 6 years. Both the flower and the tree produced in 2022. In

which year will they both produce again at the same time?

This is another application of the LCM concept. We need to find the LCM of 4 and 6 to determine when both cycles will coincide again.

Factors:
$4 = 2^2$
$6 = 2^1 \times 3^1$

Taking the maximum power of each factor:
$LCM(4, 6) = 2^2 \times 3^1 = 4 \times 3 = 12$

Since both the flower and tree produced in 2022, they will both produce again at the same time after 12 years.

Therefore, they will both produce again in 2022 + 12 = 2034.